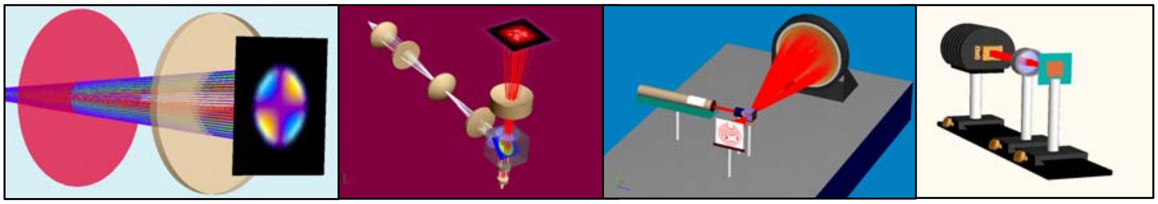
FRED®

Slit Diffraction Application Note



The classic problem of diffraction through a slit finds one of its chief applications in spectrometers. The wave nature of these phenomena can be modeled quite accurately using the complex raytracing method of Gaussian Beam Decomposition (GBD) available to FRED users. This Application Note will discuss the specifics of two common slit configurations; illumination by a plane wave and by light focused by a lens.

A clear understanding of the principles behind GBD is essential in constructing an accurate model of slit diffraction. GBD uses Gaussian beamlets as a basis set for the construction of all coherent sources. Gaussians are chosen due to the special properties inherent in their mathematical representation. Each ray in the source grid is accompanied by additional rays called secondary rays as they trace through an optical system. These secondary rays represent the “waist” and “divergence” of the individual Gaussian beamlet. By virtue of being Gaussian, these beamlets obey the well-known equation relating far-field divergence half angle θ and minimum beam waist radius ω_o:

$$\tan\theta = \lambda/\pi\omega_o \qquad (1)$$

In order to effectively sample surfaces in an optical model, these beamlets must obey the paraxial approximation. This is perhaps the most important consideration in properly implementing coherent raytracing. In almost every practical case, attempting to operate outside this paraxial limit negates the ability of these Gaussian beamlets to accurately sample optical components as they propagate. Indeed, failure of secondary rays to remain well correlated with their parent ray leads to coherent ray errors and erroneous irradiance calculations.

While not precisely defined, paraxial approximation can be expressed in at least two forms; $\tan\theta \approx \theta$ and $\theta << \pi$. In both cases, a reasonable choice for θ is 0.1 radians. The direct and most obvious implication of the paraxial approximation is therefore the beamlet radius ω_o must be greater than or equal to 3λ. This relationship represents a lower limit on beamlet width. In practice, the user should thoughtfully consider operating with some amount of margin, say 5-10λ. There are likely cases in which even this conservative lower may not be adequate.

When FRED sets up a coherent grid, it uses the grid spacing and a beam overlap factor as the measure of beamlet width. The overlap factor is the fractional overlap

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between adjacent beamlets and has a default value of 1.5. Thus, for visible light (λ=0.5µm), grid spacing cannot be less than about (2/1.5)·3·0.5µm = 2µm; a more reasonable limit being closer to 5-10µm.

Slit with Plane Wave Illumination

Armed with an understanding of the paraxial constraint inherent in GBD, lets us now consider the problem of a plane wave incident upon a narrow slit. Elementary physics texts discuss this topic in terms of wave propagation for the case where d<<λ. Thus, analytic results are readily available for comparison to our *FRED* results as a check of the method's validity.

A plane wave source created in *FRED* consists of a rectilinear grid of rays all with a common direction vector. Considering the fact that a plane wave extends to infinity in directions perpendicular to its propagation direction, creating a grid of finite extent and shape amounts to having passed an infinite plane wave through an aperture whose dimensions are those of the grid dimensions. This profound yet simple realization allows the problem of plane wave diffraction through a slit to be simulated without the need for representing the slit as a physical object. Thus, this slit diffraction problem is reduced to creating a grid whose dimensions match that of the slit itself and tracing those rays to the far-field to observe the diffraction pattern.

Consider a slit of width 60µm and length 1.2mm illuminated with light at the HeNe wavelength 0.6328µm. According to the paraxial constraint, no more than about 15-20 rays can be defined across the narrow dimension. If we make a conservative choice of 11, then the long dimension of the slit source should have 220 rays to insure an even grid spacing. Figure 1 shows the irradiance pattern of this grid before tracing. Worth noting is the roll off at the edges of the irradiance patterns due to the finite width of the individual Gaussian beamlets.

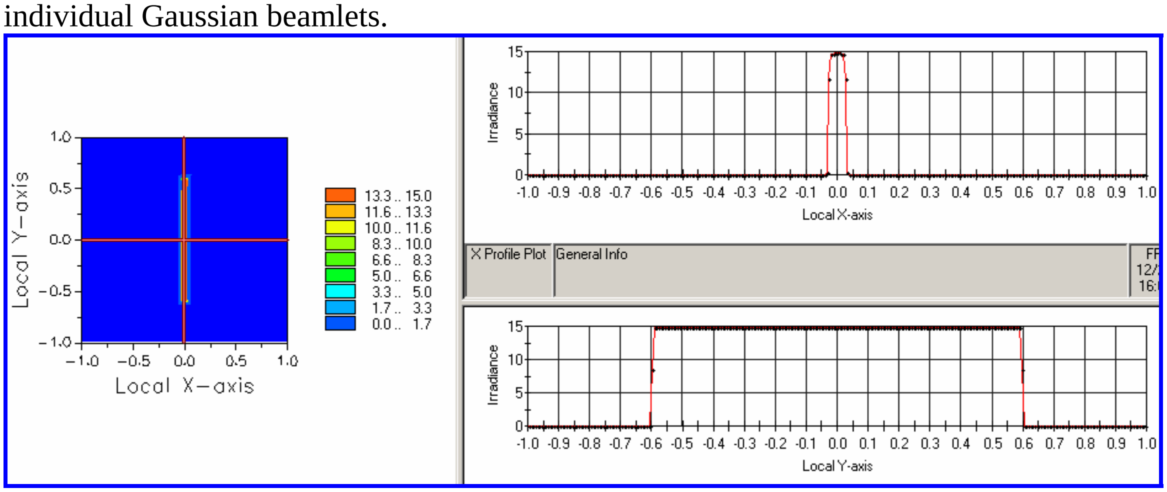

Figure 1. Irradiance pattern of slit source.

If this ray grid is now traced to a plane in the far-field some 5000mm away, an irradiance calculation yields the pattern shown in Figure 2. Based upon the analytic expression for diffraction through a slit of halfwidth a,

$$I = \left(\frac{\sin kpa}{kpa} \right)^2 I_o \qquad (2)$$

where $k=2\pi/\lambda$, and $p=\sin\theta$, the zeros occur for $\sin(\theta_n)=n\lambda/2a$ (n = 0,1,2,..).

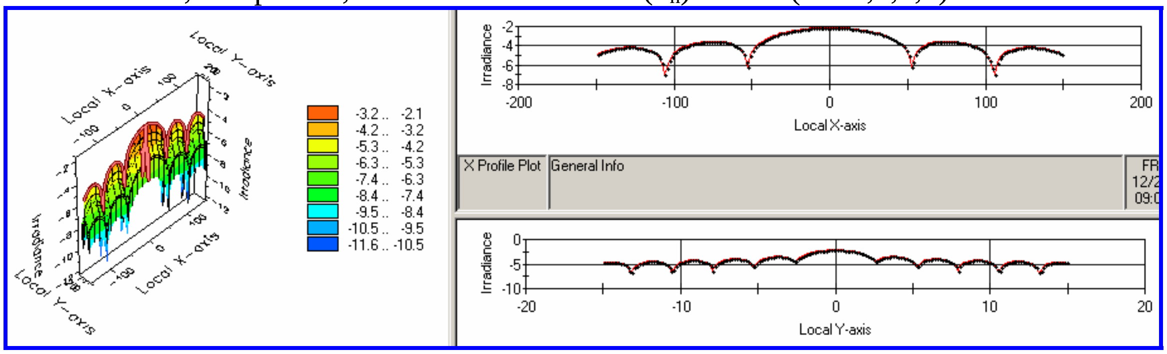

Figure 2. Log plot of irradiance 500 mm from slit.

The first zero of Eq. 2 (n=1) occurs at θ = asin(0.633/60) = 0.6°. From Figure 2, the first zero occurs at x=52.5mm which yields θ = atan(52.5/5000)=0.6° in good agreement with Eq. 2.

It is worthwhile to examine the Gaussian beamlets at the measurement plane with regard to the original choice of grid spacing using FRED's *Gaussian Ray Size Spot Diagram* feature. Having chosen beamlets on the order of tens of waves across for the original grid, these beamlets have diverged to more than 400mm in diameter in traversing the 5000mm as shown in Figure 3. As a result, the beamlets would be able to properly sample only optical elements whose functional representation both extends beyond these limits and are no more than quadratic over the same region. Indeed, this is a severe restriction limiting the element type to planes and very large radii spheres.

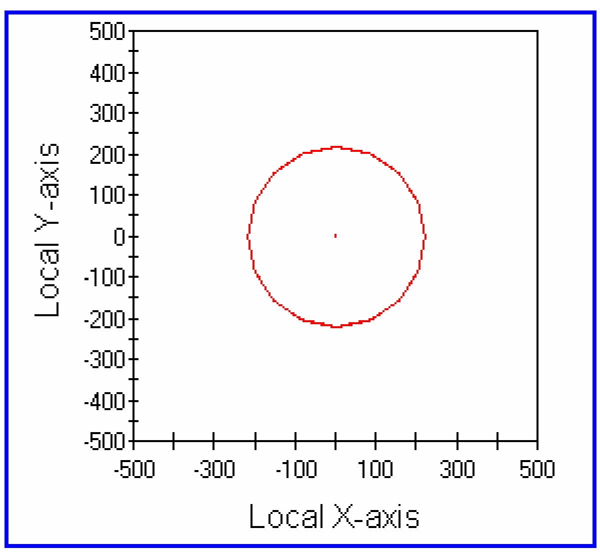

Figure 3. Gaussian Beam Size Spot Diagram

Slit Illuminated with Focused Beam

Illumination of a slit with a focused beam represents a fundamentally different problem than the previous plane wave example in the context of complex raytracing. It is not sufficient to simply trace a focused bundle of rays through the slit in an effort to simulate its diffraction effects. This point offers an excellent opportunity to introduce another important assumption upon which complex raytracing is based; all secondary rays associated with a base ray must intersect the same object as the base ray. This point has far-reaching consequences and is best illustrated by considering two trivial cases involving an aperture. If the base ray passes through an aperture then all its associated secondary rays pass and conversely, if a base ray is blocked then all its associated secondary rays are blocked as well. For a further discussion on this topic, see *FRED*'s Help document under the topic *Coherent Source Introduction*.

Consider now a parabolic reflector which, when used on-axis, produces an RMS spot size equal to zero. All rays pass through a single point in space. Of course, the diffraction spot or point spread function is an Airy disk whose radius is given by 1.22λf/#. Even if the slit width were chosen equal to a fraction of this radius, all the base rays would pass unobstructed through the slit. *FRED* would act as if the slit were non-existent.

Coherent Field Synthesis as implemented in *FRED* offers a method for addressing phenomena such as diffraction from a slit illuminated with a focused beam or the more common spatial filtering of a beam through a pinhole. This process involves computing the complex scalar field in the aperture plane and trimming the field according to the aperture geometry. The original rayset is then replaced by a rayset synthesized from the trimmed field.

Let us now examine the procedure required to simulate 10μm light focused with an f/2.5 lens onto the slit used in the previous example. This layout is shown in Figure 4. While a slit plane is included, the slit itself is not part of this model. Its presence is introduced indirectly by appropriate trimming of the complex field in the *Coherent Field Synthesis* dialog.

Figure 4. Germanium lens focusing a plane wave at f/2.5.

After tracing to the slit plane, *FRED* is instructed to compute the complex field using its *Coherent Scalar Field* feature. This calculation is shown in Figure 5. The field is then saved to a text file allowing import into the *Coherent Field Synthesis* dialog shown in Figure 6. Truncation of the field according to the slit dimensions is accomplished by editing cells in the spreadsheet using the Modify Field Values tool.

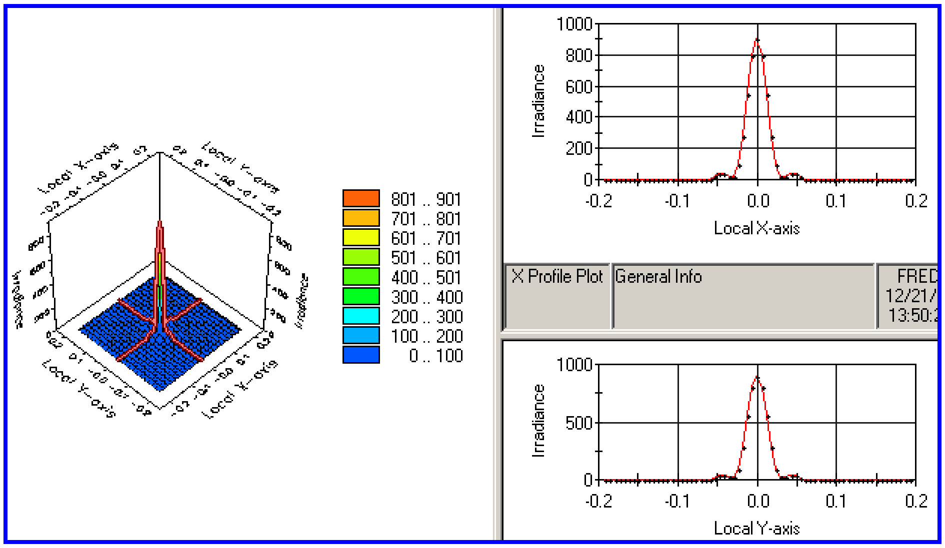

Figure 5. Focused spot before truncation.

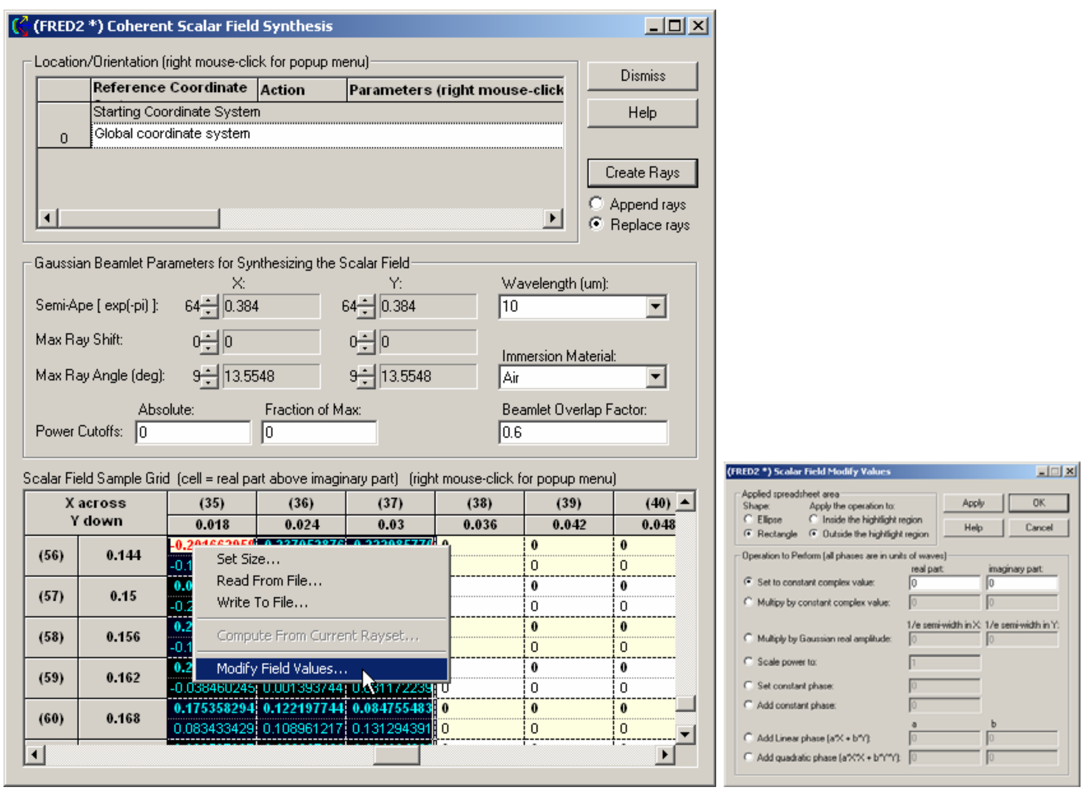

Figure 6. Coherent Field Synthesis dialog with field truncation per slit dimensions.

To complete the synthesis process, several of the Gaussian Beamlet Parameters in the middle section of the dialog box must be adjusted. The user will note that there are a range settings for beamlet semi-aperture, rays shift and ray angle. These settings offer the flexibility required to control how the new rayset can sample downstream optical elements. By setting the Semi-Ape to its maximum value, all the beamlets acquire a common origin and a size comparable to the entire Analysis Surface area. As a result, each beamlet will originate at the center of the Analysis Surface area with a different direction, amplitude and phase. This choice of maximum semi-aperture also insures that the beamlets remain well within the paraxial approximation. The Max Ray Angle determines the outer envelope of this ray fan which need not be greater than the angle subtended by the next optical element. If another lens identical to the first is added to the layout and located at twice its focal length from the slit plane, then the maximum angle needed is then approximately 14°. Figure 7 shows the field at the slit plane after resynthesis.

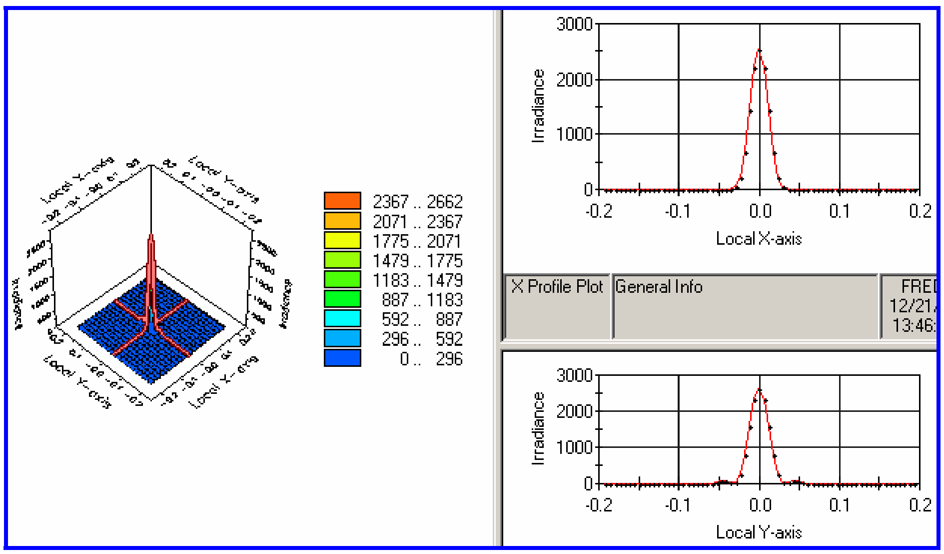

Figure 7. Truncated field calculated from the new rayset.

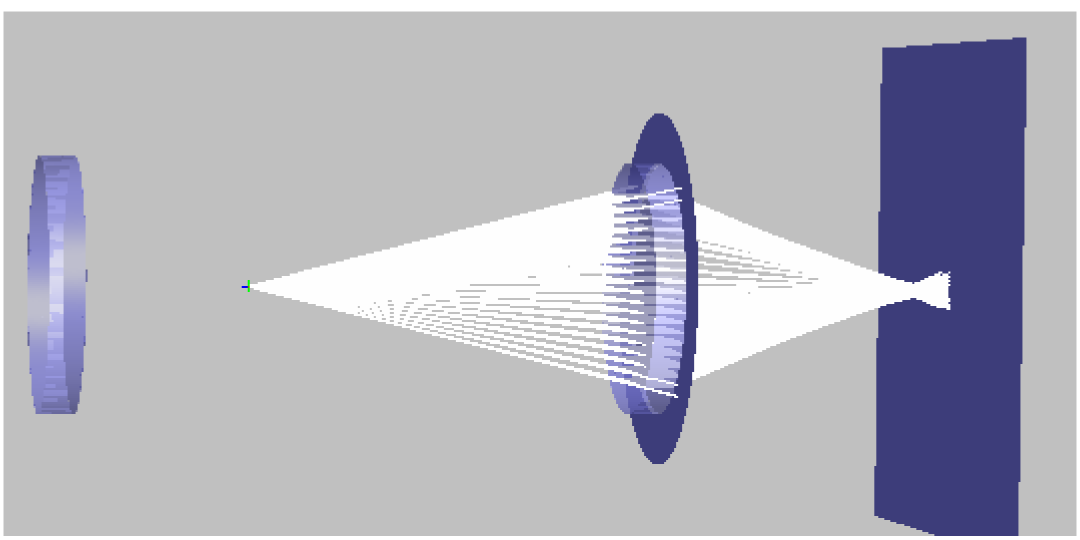

Figure 8. Synthesized rayset traced to image plane of focusing lens.

When the newly synthesized rayset is traced from its origin and imaged by the second lens, there is clear evidence in the irradiance calculation of the beam having been apertured; the higher frequencies are significantly suppressed only in the x-dimension as shown in Figure 9. Note that a similar example involving spatial filtering with a circular pinhole can be found in FRED's Help under the *Coherent Field Synthesis* topic.

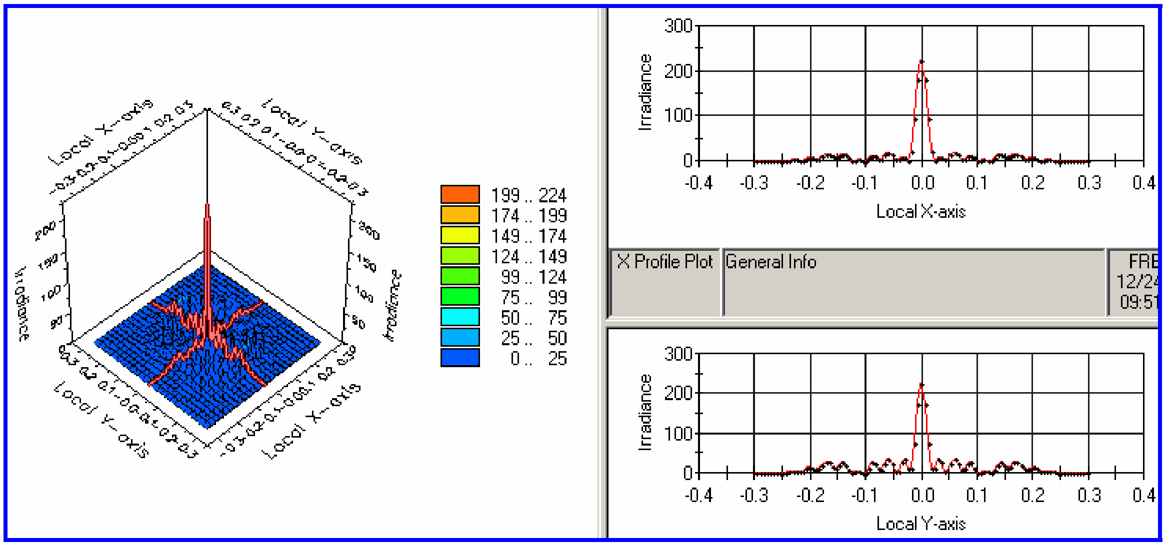

Figure 9. Irradiance at focus of second lens.